

DJ Trigonometry A Square-Based Proportional Framework for Closed Geometric Figures

Dheeraj Badwan, Shahid Bhagat Singh Government College, Ashta, Madhya Pradesh, India

ORCID: <https://orcid.org/0009-0005-7836-4204>

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Abstract

This research introduces DJ Trigonometry, a new and independent trigonometric framework developed to analyze angular and proportional relationships in closed and polygon-based geometric figures without reliance on circular geometry. Classical trigonometry is grounded in circles, radians, and trigonometric functions such as sine, cosine, and tangent, which often lead to conceptual abstraction and learning difficulties. To address these limitations, the proposed framework shifts the geometric foundation from circular to square-based structures. The core concepts of DJ Trigonometry include DJ Angle, DJ Degree, and DJ Measure, which together establish a direct proportional relationship between angular measurement and linear dimensions. A specialized measuring instrument, the DJ Protractor, is introduced to determine DJ Angles within polygonal geometries. Unlike classical angle systems, the DJ Degree represents a complete square-based rotational cycle and is geometrically distinct from circular rotation. A major theoretical contribution of this work is the DJ Badwan Theorem, which provides a direct proportional method for determining side lengths in a right-angled triangle by multiplying the adjacent side by the DJ Measure of the given acute angle. This approach eliminates the need for classical trigonometric functions and tables, relying on simple arithmetic and proportional reasoning. The framework is extended to determine hypotenuse relationships using the Pythagorean theorem within the DJ system. By redefining angular measurement through square-oriented geometry, DJ Trigonometry offers a simplified, intuitive, and pedagogically accessible alternative to classical trigonometry. The proposed system does not replace traditional methods but complements them by providing a new perspective for the study of closed geometric figures, with applications in mathematics education and conceptual geometry.

Keywords: DJ Trigonometry, DJ Measure, DJ Degree, DJ Angle, DJ Protractor, DJ Badwan Theorem.

1. Introduction

Trigonometry is a fundamental branch of mathematics that studies the relationship between angles and sides of geometric figures, especially triangles. Classical trigonometry is primarily based on circular geometry, where angles are measured using circular arcs and expressed in degrees or radians. Trigonometric functions such as sine, cosine, and tangent arise from this circular framework and are widely used in geometry, physics, astronomy, and engineering. Although classical trigonometry has strong theoretical foundations, it is often considered abstract and difficult for learners due to its dependence on circular reasoning, trigonometric functions, and symbolic representations. Furthermore, its geometric interpretation is largely limited to circular and rotational systems, making the analysis of polygonal and

closed figures less intuitive. To overcome these limitations, this research introduces DJ Trigonometry, a new and alternative trigonometric framework specifically developed for closed, polygon-based geometric figures such as triangles, squares, rectangles, cubes, and cuboids. Unlike conventional trigonometry, DJ Trigonometry does not rely on circular arcs, radians, or π . Instead, it is founded on square-based geometry and proportional relationships.

The central concept of this system is the DJ Measure, along with a newly defined angular unit known as the DJ Degree. These quantities provide a direct proportional link between angular measurement and linear dimensions in closed figures. To practically implement this measurement system, a specialized instrument called the DJ Protractor is introduced. This protractor is designed exclusively for measuring DJ Angles and visualizing angular relationships within polygonal geometries. A key theoretical contribution of DJ Trigonometry is the DJ Badwan Theorem, which establishes a direct proportional relationship between the sides of a right-angled triangle using the DJ Measure. This theorem enables the determination of one side of a triangle directly from another without employing classical trigonometric functions such as sine, cosine, or tangent. The DJ Badwan Theorem provides an independent and conceptually simple geometric interpretation of angle-side relationships within the DJ framework. By shifting the foundation of trigonometry from circular geometry to polygonal geometry, DJ Trigonometry offers a simplified, intuitive, and proportional approach to geometric analysis. This framework is particularly suited for educational and practical applications where clarity, direct measurement, and geometric intuition are essential. Thus, DJ Trigonometry presents a modern reinterpretation of trigonometric principles, offering a new perspective for understanding angular and linear relationships in closed geometric structures through the DJ Measure and the DJ Badwan Theorem.

2. Literature Review

Trigonometry has been a central area of mathematics since ancient times. In Euclid's Elements, the principles of proportion, similarity, and geometric construction were established, which later became the foundation of trigonometry. Classical trigonometry is primarily based on circular geometry, where angles are defined using arcs, radii, degrees, and radians.

In traditional trigonometry, functions such as sine, cosine, and tangent play a fundamental role. Historians of mathematics such as Boyer and Merzbach, Katz, and Maor have shown that this form of trigonometry evolved largely from astronomical calculations, circular motion, and wave-related phenomena. Standard textbooks such as Thomas' Calculus and Stewart's Calculus also present trigonometry predominantly within a circular framework. However, several scholars have pointed out that circle-based angle measurement systems often become abstract for learners, especially when trigonometry is applied to polygonal or closed geometric figures. Courant and Robbins emphasized that the more direct and visual a geometric interpretation is, the stronger conceptual understanding becomes. Similarly, Posamentier and Polya highlighted proportional reasoning as a powerful tool in the teaching and understanding of geometry. Although significant work has been carried out on polygonal geometry and square-based structures, attempts to redefine angular measurement and trigonometric relationships entirely within a non-circular framework remain limited. Traditional degree and radian systems are most effective for circles, cones, and spherical figures. However, no independent and simplified measurement system has been firmly

established for understanding angle-side relationships in polygons and closed linear figures. It is within this context that DJ Trigonometry offers a novel and independent perspective. This system is not dependent on circular arcs, π , or radians, but is instead grounded in square-based geometry and proportional relationships. The concepts of DJ Measure, DJ Degree, and DJ Angle are geometrically consistent while being fundamentally distinct from classical trigonometric definitions. In this sense, DJ Trigonometry practically fulfills the need for intuitive geometric interpretation emphasized by scholars such as Needham and Stillwell.

In particular, the DJ Badwan Theorem emerges as an independent proportional principle rather than a replacement for the classical tangent function. The theorem demonstrates that, in a right-angled triangle, the relationship between the sides can be established directly through the DJ Measure, enabling simplified and more intuitive calculations. According to the educational principles advocated by the National Council of Teachers of Mathematics (NCTM), such alternative frameworks enhance conceptual clarity and accessibility in mathematics learning. Thus, the existing literature indicates that while classical trigonometry is a mature and well-established discipline, there remains a clear need for an independent, non-circular trigonometric framework suitable for closed and polygon-based geometric figures. DJ Trigonometry addresses this research gap and contributes a new conceptual approach to the field of trigonometric analysis.

3. The DJ Trigonometric Framework

3.1 Measurement of DJ Angles Using the DJ Protractor

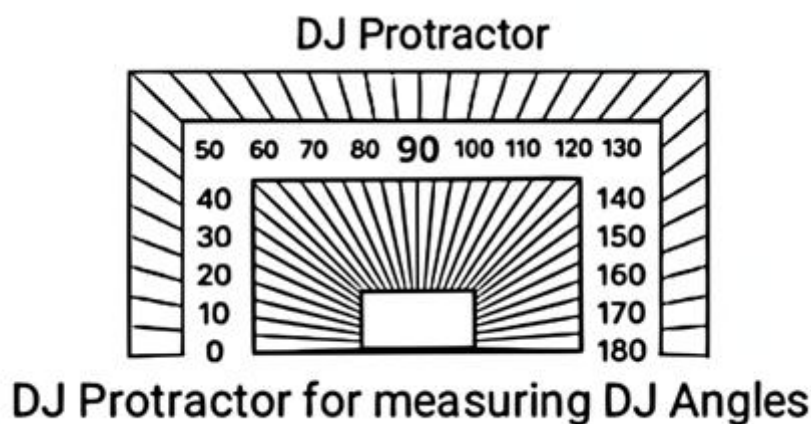


Figure 1. DJ Protractor for Measuring DJ Angles

The DJ Protractor is a specially proposed geometric instrument designed for measuring DJ Angles as defined within the DJ Trigonometric framework. This instrument differs from the traditional circular protractor because it is not based on circular geometry but on square-based geometry. The DJ Protractor is constructed in accordance with the concepts of DJ Degree and DJ Measure, enabling the representation of angular positions without the use of circular arcs, radians, or classical trigonometric functions. The structural design of the DJ Protractor is founded on the principle of square-based rotation, in which a complete square rotation is considered to be divided into 360 DJ Degrees. This structure enables consistent and proportional measurement of DJ Angles in closed and polygonal geometric figures and remains fully compatible with the theoretical framework of DJ Trigonometry. The method of using the DJ Protractor is defined as follows. The point at which the DJ Angle is to be measured is

placed at the center of the DJ Protractor. One arm of the DJ Angle is aligned with a selected standard radius, referred to as the reference radius. Subsequently, the second arm of the DJ Angle extends toward a specific radius or a DJ Degree marking indicated on the protractor. This marking represents the DJ Degree value of the given DJ Angle. From the obtained DJ Degree value, the corresponding DJ Measure can be directly determined using the conversion relationships defined within the DJ Trigonometric system.

Thus, the DJ Protractor may be regarded not only as an instrument for measuring angles but also as a means of clearly establishing the proportional relationship between a DJ Angle and its arms. This proportional representation forms the basis for further geometric and trigonometric calculations within DJ Trigonometry.

3.2 DJ Angle

A DJ Angle is an angular quantity defined within the DJ Trigonometric framework in accordance with square-based geometry. It represents the relative positional orientation of two arms about a point; however, unlike classical angles, its representation is not based on circular geometry but on the principle of square-based rotation. The measurement of a DJ Angle is expressed in terms of DJ Degree or DJ Measure, both of which are independently defined within DJ Trigonometry. A complete square-based rotation is considered to be divided into 360 DJ Degrees, and the measure of a DJ Angle is determined accordingly. This measurement system is not mathematically equivalent to the classical degree or radian systems, but instead represents an independent angular framework. The determination of a DJ Angle is carried out using the DJ Protractor, a specialized geometric instrument developed in accordance with square-based structural principles. The value of a DJ Angle can be further converted into DJ Measure, which is used to establish proportional relationships among the sides of right-angled triangles and other closed geometric figures. Thus, the DJ Angle constitutes a fundamental concept of DJ Trigonometry, providing a proportional representation of angular position and offering an alternative geometric perspective distinct from classical trigonometric approaches.

3.3 DJ Degree Measure

A DJ Degree is an angular unit of measurement used within the DJ Trigonometric framework for measuring DJ Angles. This unit is based on square-based rotational geometry and is distinct from the traditional circular degree system. In DJ Trigonometry, a complete square-based rotation is considered to be divided into 360 equal parts, each of which is defined as one DJ Degree. Accordingly, the DJ Degree provides a numerical representation of the square-based rotational position of a DJ Angle. It is explicitly emphasized that the DJ Degree is not used as a mathematical equivalent of the classical degree or radian measurement systems. Although the numerical value of 360 is adopted, the underlying geometric foundation is square-based rather than circular. Therefore, the meaning, representation, and application of the DJ Degree are fundamentally different from those of conventional angular units.

The DJ Degree is employed in the measurement of DJ Angles using the DJ Protractor and is further converted into DJ Measure to establish proportional relationships among the sides of right-angled triangles and other closed geometric figures.

Thus, the DJ Degree constitutes a fundamental angular unit of DJ Trigonometry, providing an independent and consistent framework for angular measurement within

square-based geometry. If the rotation from the initial side to the terminal side is one three-hundred-sixtieth ($1/360$) of a complete revolution, then the DJ Angle is said to have a measure of one DJ Degree, written as 1 DJ Degree. One DJ Degree is divided into 60 minutes, and one minute is further divided into 60 seconds. One sixtieth of a DJ Degree is called a minute and is written as $1'$, and one sixtieth of a minute is called a second and is written as $1''$.

Thus, $1 \text{ DJ Degree} = 60'$ and $1' = 60''$.

3.4 DJ Measure

A DJ Measure is a proportional quantity defined within the DJ Trigonometric framework that expresses the magnitude of a DJ Angle in relation to the linear dimensions of a geometric configuration. Unlike classical trigonometric ratios such as sine, cosine, or tangent, DJ Measure is derived from square-based rotational geometry and does not rely on circular arcs or traditional trigonometric functions. In DJ Trigonometry, the square-based rotational system divides a complete rotation into 360 DJ Degrees. The DJ Measure of an angle is obtained by relating the position of the angle's terminal arm, as determined using the DJ Protractor, to the corresponding DJ Degree value. This DJ Degree value is then translated into a DJ Measure through the defined conversion relationships within the DJ system. Mathematically, DJ Measure represents a ratio between the effective proportional extension of a geometric side relative to a standard reference within a square-based structure. It serves as a direct proportional parameter for establishing relationships among the sides of right-angled triangles and other closed geometric figures without resorting to classical trigonometric ratios or tables. The use of DJ Measure enables the representation of angular magnitude and its associated linear relationships in a form that is consistent with the square-oriented conceptual basis of DJ Trigonometry. It provides a coherent framework for analyzing geometric configurations where proportional reasoning replaces circular trigonometric functions. Thus, DJ Measure functions as a fundamental component of DJ Trigonometry, facilitating the translation of angular orientations into quantifiable proportional relationships suitable for geometric analysis and application.

Radius of a Square

If the side length of a square is a , then the radius r of the square is given by

$$r = a / 2$$

$$\text{Perimeter of the square} = (\text{DJ Measure}) \times r.$$

Another unit for measuring a DJ Angle is the DJ Measure. In a unit square of radius 1 unit, if a segment of the square perimeter of length 1 unit subtends a DJ Angle at the centre, then the measure of that DJ Angle is defined as 1 DJ. We know that the equivalent complete rotational boundary of a square of radius 1 unit is 8 units. Therefore, one complete rotation of the initial side produces a DJ Angle of 8 DJ. More generally, if a square has a radius of r units, then a segment of the square perimeter of length r units subtends a DJ Angle of 1 DJ at the centre. Equal segments of the square perimeter subtend equal DJ Angles. Hence, if in a square of radius r , a segment of the square perimeter of length l units will subtend a DJ Angle of l/r DJ at the centre. Thus, if in a square of radius r , a segment of the square perimeter of length l subtends a DJ Angle θ DJ at the centre, then the relation is given by $\theta = l/r$ in DJ Measure.

$$\text{DJ Measure} = (\text{Perimeter of the square}) / (\text{radius of the square})$$

$$\text{DJ Measure} = (\text{Perimeter of the square}) / (\text{radius of the square})$$

3.5 Relation between DJ Degree and DJ Measure

Since a square subtends a DJ Angle at the centre whose DJ Measure is 8, and its DJ Degree Measure is 360 DJ Degree, it follows that:

$$8 \text{ DJ} = 360 \text{ DJ Degree}$$

or

$$4 \text{ DJ} = 180 \text{ DJ Degree}$$

The above relation enables us to express a DJ Measure in terms of DJ Degree Measure and a DJ Degree Measure in terms of DJ Measure.

Using this relation, we obtain:

$$\begin{aligned} 1 \text{ DJ} &= 180 / 4 \text{ DJ Degree} \\ &= \text{approximately } 45 \text{ DJ Degree, Also,} \\ 1 \text{ DJ Degree} &= 4 / 180 \text{ DJ} \\ &= \text{approximately } 0.022 \text{ DJ} \end{aligned}$$

Thus, the relation between DJ Degree Measures and DJ Measures of some common DJ Angles is given in the following table:

DJ Degree						
30 DJ Degree	45 DJ Degree	60 DJ Degree	90 DJ Degree	180 DJ Degree	270 DJ Degree	360 DJ Degree
DJ Measure						
4/6	4/4	4/3	4/2	4	6	8

Table 1. Relation between DJ Degree and DJ Measure.

Notational Convention.

Since DJ Angles are measured either in DJ Degrees or in DJ Measure, we adopt the following notational convention. Whenever we write a DJ Angle θ DJ Degree, it means the DJ Angle whose DJ Degree Measure is θ . Similarly, whenever we write a DJ Angle β , it means the DJ Angle whose DJ Measure is β . It is important to note that when a DJ Angle is expressed in DJ Measure, the term "DJ Measure" is usually omitted and is understood from the context. For example, the quantities 4 and 4/4 are often written without explicitly mentioning "DJ Measure", and with this understanding,

$$4 \text{ DJ Measure} = 180 \text{ DJ Degree}$$

and

$$4/4 \text{ DJ Measure} = 45 \text{ DJ Degree.}$$

Hence, we obtain the following conversion relations:

$$\text{DJ Measure} = (4 / 180) \times \text{DJ Degree Measure}$$

$$\text{DJ Degree Measure} = (180 / 4) \times \text{DJ Measure}$$

DJ Measure = (4 / 180) × DJ Degree Measure
DJ Degree Measure = (180 / 4) × DJ Measure

Thus, using the above relations, one can easily convert the DJ Degree Measure of any DJ Angle into its DJ Measure, and vice versa.

3.6 DJ Acute Angle

A DJ acute Angle is defined as any DJ Angle less than 90 DJ Degree. It represents

an angle smaller than a right angle within the DJ measurement system. The DJ acute angle is composed of two fundamental parts, namely the D acute angle and the J acute angle, which together form the complete structure of the right-angled relationship in DJ trigonometry.

3.6.1 D Acute Angle

A D acute Angle is defined as 45 DJ Degree or any DJ Angle less than 45 DJ Degree. This type of angle represents the smaller range of acute measurements within the DJ system, focusing on sharp, narrow angular formations. The D acute angle symbolizes the lower spectrum of acute measurements and plays a significant role in the proportional and symmetrical understanding of closed geometrical figures.

3.6.2 J Acute Angle

A J acute angle is defined as 45 DJ Degree or any DJ Angle greater than 45 DJ Degree but less than 90 DJ Degree. This represents the broader portion of the acute range, where the angle extends closer to the right angle while maintaining its acute nature. The J acute angle thus complements the D acute angle to complete the total range of acute measurements within the DJ system.

Sum of DJ Angles in a Triangle

According to DJ Trigonometry, the sum of the three DJ Angles present in any triangle is equal to 180 DJ Degree. If the triangle is a right DJ Angle triangle, then one of its angles is a right DJ Angle, whose measure is 90 DJ Degree. Therefore, in a right DJ Angle triangle, the sum of the remaining two angles, namely the D acute angle and the J acute angle, is obtained by subtracting the right angle from 180 DJ Degree. Thus,

$$\text{D acute angle} + \text{J acute angle} = 180 \text{ DJ Degree} - 90 \text{ DJ Degree}$$

That is,

$$\text{D acute angle} + \text{J acute angle} = 90 \text{ DJ Degree}$$

$$\text{D acute angle} + \text{J acute angle} = 90 \text{ DJ Degree}$$

Hence, it is established that in any right DJ Angle triangle, the sum of the D acute Angle and the J acute angle is always equal to 90 DJ Degree.

4. DJ Badwan Theorem

4.1 Statement of the Theorem

DJ Badwan Theorem states that within the DJ Trigonometric Framework, the relationship between the sides of a right-angled triangle corresponding to a given DJ acute angle is governed by direct proportionality rather than circular trigonometric functions. In a right-angled triangle, if the DJ Measure of a given D acute angle is known, then the length of the side opposite to that angle can be obtained directly by multiplying the length of the side adjacent to the angle by the DJ Measure of the angle. The essential requirement in DJ Trigonometry is that: The side adjacent to the D Acute Angle is considered as the radius of the square, and the side adjacent to the J Acute Angle is considered as the perimeter-related proportional side of the square. Hence, in DJ Trigonometry, the determination of triangle sides is governed exclusively by their adjacency to the D Acute Angle and the J Acute Angle, rather than by conventional geometric orientation. This square-based interpretation allows DJ Trigonometry to treat the triangle as a closed geometric figure, where angle-side relationships are expressed in a direct, proportional, and non-circular manner. As a result, the framework eliminates the necessity of classical trigonometric notions such as sine, cosine, tangent, and circular geometry, and instead relies on simple linear proportionality derived from

square-oriented geometry. This theorem establishes a simple, linear, and square-based proportional rule for determining side lengths without the use of sine, cosine, tangent, radians, or circular geometry.

4.2 Mathematical Formulation

Let the DJ Measure of a D acute angle be x . The DJ Measure is defined as the ratio of the Perimeter of the square to the radius of the imaginary square.

$$\text{DJ Measure} = (\text{Perimeter of the square}) / (\text{radius of the square})$$

Let O be the length of the side adjacent to the D acute angle. This side is also interpreted as the radius of the imaginary square.

Let P be the length of the side opposite to the D acute angle. This side is also interpreted as the perimeter of the imaginary square.

According to the DJ Badwan Theorem: in a right-angled triangle, if the D acute angle is given, then the length of the side opposite to the D acute angle (P), or equivalently the perimeter of the imaginary square, can be obtained by multiplying the length of the side adjacent to the D acute angle (O), or equivalently the radius of the imaginary square, by the DJ Measure (x) of that angle. Mathematically,

$$(x) = (4/180) \text{ D acute angle}$$

Furthermore, the value of P is always less than or equal to the value of O

$$\text{DJ Measure of the D acute angle} = x$$

$$\text{DJ Measure} \times (\text{radius of the square}) = (\text{Perimeter of the square})$$

$$\text{Length of the adjacent side of the D acute angle} = O = (\text{radius of the square})$$

$$\text{Length of the side opposite to the D acute angle} = P = (\text{Perimeter of the square})$$

$$P = O \times x$$

$$O = P/x$$

$P = O \times x$
$O = P/x$

Where:

= the side opposite the given acute angle

→ It can be the base or the perpendicular, depending on the position.

= the side adjacent to the given acute angle

→ It can be the base or the perpendicular, depending on the position.

The result obtained from this multiplication represents the actual length of the side opposite to the D acute angle. In other words, when the adjacent side is multiplied by the DJ Measure of the D acute angle, the obtained value is equal to the length of the side opposite to that angle.

4.3 Proof / Logical Justification

In DJ Trigonometry, angular measurement is not derived from circular arcs but from square-based geometry. The DJ Measure is defined as the ratio of the effective square boundary length to its corresponding radius. Consider a right-angled triangle in which the adjacent side to the D acute angle is treated as the radius of an imaginary square. The opposite side is interpreted as the proportional square boundary corresponding to that angle. Because equal DJ Angles subtend proportional square boundary lengths, the ratio of the opposite side to the adjacent side must be equal to the DJ Measure of the angle.

4.4 Comparative Clarification of DJ Trigonometry

It is necessary to explicitly clarify that all concepts used in DJ Trigonometry are not equivalent to the corresponding concepts of classical or traditional trigonometry. Rather, they are independent, distinct, and founded on a new geometric basis.

DJ Angle – The DJ Angle is not equivalent to the classical angle. Classical angles are fundamentally based on circular geometry, whereas the DJ Angle is defined within a square-based geometric framework. Consequently, their measurement, interpretation, and application differ at a fundamental level.

DJ Degree – Although the DJ Degree visually resembles the classical degree system by adopting the number 360, it is not mathematically equivalent. Classical degrees arise from circular rotation, while DJ Degree are derived from a square-based rotational cycle. Therefore, the geometric meaning of a DJ Degree is fundamentally different from that of a classical degree.

DJ Measure – The DJ Measure is not a reformulation of the radian measure. In classical trigonometry, the radian is defined as the ratio of arc length to radius in a circle. In contrast, the DJ Measure is derived from proportional relationships involving the perimeter and linear dimensions of a square. Hence, DJ Measure represents a polygon-based proportional quantity rather than a circular one.

DJ Protractor – The DJ Protractor is not a modified version of the traditional protractor. It is a newly designed geometric instrument developed specifically for measuring DJ Angles. While the classical protractor operates on circular angle measurement, the DJ Protractor functions according to square-based and polygonal geometry.

DJ Acute Angle – The DJ Acute Angle is conceptually different from the classical acute angle. In DJ Trigonometry, an acute angle is defined within a square-based framework and is further divided into two components: the D acute angle and the J acute angle. Such a structural division does not exist in classical trigonometry and arises naturally from the DJ geometric system.

DJ Badwan Theorem – The DJ Badwan Theorem is not equivalent to the classical trigonometric function $\tan \theta$. Although both involve a proportional relationship between sides of a right-angled triangle, their conceptual foundations and definitions are fundamentally different. The DJ Badwan Theorem is entirely based on DJ Degree and DJ Measure, which are independent of circular geometry and trigonometric functions. It employs direct proportional multiplication rather than functional evaluation.

DJ Formulae – All formulae used in DJ Trigonometry are distinct from those of classical trigonometry. In the DJ framework, relationships between angles and sides are expressed through linear proportional relationships instead of sine, cosine, or tangent functions.

Therefore, it is established that DJ Trigonometry is neither a transformation nor a replacement of classical trigonometry. Instead, it constitutes an independent, square-based, non-circular trigonometric framework designed to analyze angular and linear relationships in closed geometric figures such as triangles, squares, rectangles, and other polygonal structures.

5. Derived Relationships

5.1 Hypotenuse-Adjacent Side Relationship

Conceptual Proportional Relationship between the Hypotenuse (K) and the Adjacent Side (O) in DJ Trigonometry. By Pythagoras theorem, in a right-angled triangle,

the square of the hypotenuse is equal to the sum of the squares of the base and the perpendicular. Hence,

$$K^2 = O^2 + P^2$$

Here,

K denotes the hypotenuse of the right-angled triangle.

O denotes the base, which is the adjacent side to the given D acute angle.

P denotes the perpendicular, which is the opposite side to the same D acute angle.

Thus, in DJ Trigonometry, O is always taken as the side adjacent to the D acute angle, and P is always taken as the side opposite to the D acute angle.

According to DJ Trigonometry, the DJ Measure is defined as the ratio of the opposite side to the adjacent side corresponding to the D acute angle. Therefore,

$$\text{DJ Measure} = x = P / O$$

Further, the DJ Measure is related to the DJ Degree through the D acute angle as,

$$\text{DJ Measure} = (4 / 180 \text{ DJ Degree}) \times (\text{D acute angle})$$

Now, dividing the Pythagoras equation by O^2 , we obtain,

$$K^2 / O^2 = O^2 / O^2 + P^2 / O^2$$

$$K^2 / O^2 = 1 + (P / O)^2$$

$$\text{Since } P / O = \text{DJ Measure,}$$

$$K^2 / O^2 = 1 + (\text{DJ Measure})^2$$

Taking square root on both sides,

$$K / O = \sqrt{1 + (\text{DJ Measure})^2}$$

Hence, the relationship between the hypotenuse and the adjacent side (base) corresponding to the D acute angle is,

$$K = O \sqrt{1 + (\text{DJ Measure})^2}$$

From this expression, the adjacent side (base) can be written in terms of hypotenuse and DJ Measure as,

$$O = K / \sqrt{1 + (\text{DJ Measure})^2}$$

$K = O \sqrt{1 + (\text{DJ Measure})^2}$
$O = K / \sqrt{1 + (\text{DJ Measure})^2}$

5.2 Hypotenuse–Opposite Side Relationship

Conceptual Proportional Relationship between the Hypotenuse (K) and the Opposite Side (P) in DJ Trigonometry. Now again starting from the Pythagoras theorem,

$$K^2 = O^2 + P^2$$

Dividing the entire equation by P^2 ,

$$K^2 / P^2 = O^2 / P^2 + P^2 / P^2$$

$$K^2 / P^2 = O^2 / P^2 + 1$$

Since DJ Measure = P / O , it follows that,

$$O / P = 1 / (\text{DJ Measure})$$

Substituting this value,

$$K^2 / P^2 = 1 / (\text{DJ Measure})^2 + 1$$

Taking square root on both sides,

$$K / P = \sqrt{1 + 1 / (\text{DJ Measure})^2}$$

Thus, the relationship between the hypotenuse and the perpendicular (opposite side to the D acute angle) is,

$$K = P \sqrt{(1 + 1 / (\text{DJ Measure})^2)}$$

Rearranging the above expression, the perpendicular can be expressed as,

$$P = K (\text{DJ Measure}) / \sqrt{(1 + (\text{DJ Measure})^2)}$$

$$K = P \sqrt{(1 + 1 / (\text{DJ Measure})^2)}$$

$$P = K (\text{DJ Measure}) / \sqrt{(1 + (\text{DJ Measure})^2)}$$

6. Applications / Examples

6.1 Real-life Example 1

(i) Example: Determination of the Height of a Minaret Using DJ Trigonometry.

Suppose the height of a minaret standing on a level ground is to be determined. A suitable point is selected on the ground from where the top of the minaret is clearly visible. In this situation, the minaret, the ground, and the line of sight together form a right-angled triangle. At the selected point on the ground, the DJ Angle is measured using a DJ Protractor. The base of the DJ Protractor is aligned with the horizontal ground line, and its line of sight is directed toward the top of the minaret. The angular value obtained on the DJ Protractor represents the DJ Angle for this observation.

Let the measured DJ Angle be 30 DJ Degree. This value is used for further calculations. Assume that the horizontal distance from the observation point to the base of the minaret, that is, the adjacent side of the right-angled triangle, is

$$O = 15 \text{ meters.}$$

The height of the minaret corresponds to the opposite side P of the triangle. According to DJ Trigonometry, the DJ Measure corresponding to a given DJ Angle is obtained using the relation:

$$\text{DJ Measure} = (1/45 \text{ DJ Degree}) \times \text{DJ Angle}$$

Substituting the given value,

$$\text{DJ Measure} = (1/45 \text{ DJ Degree}) \times 30 \text{ DJ Degree} = 2/3$$

According to the DJ Badwan principle, the relationship between the opposite side and the adjacent side of a right-angled triangle is given by:

$$P = O \times \text{DJ Measure}$$

Substituting the known values,

$$P = 15 \times 2/3 = 30/3 \approx 10 \text{ meters}$$

Thus, using the DJ Angle measured with the DJ Protractor and the corresponding DJ Measure, the height of the minaret is found to be approximately 10 meters. Throughout this calculation, no trigonometric functions such as sine, cosine, or tangent, and no circular concepts such as radians or π are used. The result is obtained solely through DJ Angle, DJ Degree, and DJ Measure using simple arithmetic operations.

This example clearly demonstrates that DJ Trigonometry provides a simple, direct, and practical method for determining the height of structures such as minarets, buildings, poles, or trees in real-world situations.

6.2 Real-life Example 2

(ii) Example: Determination of the Hypotenuse (K) Using DJ Trigonometry.

Suppose a right-angled triangle is formed on a level ground. From a fixed point, a DJ Angle is made toward an object, and the length of the adjacent side corresponding to this angle is known. The objective is to determine the hypotenuse of the triangle.

First, the DJ Angle is measured using a DJ Protractor from the observation point toward the object. Let the measured angle be

DJ Angle = 30 DJ Degree.

Now assume that:

Adjacent side (O) = 15 meters.

According to DJ Trigonometry, the DJ Measure is first obtained from the given DJ Angle.

$$\text{DJ Measure} = (1/45 \text{ DJ Degree}) \times \text{DJ Angle}$$

$$\text{DJ Measure} = (1/45 \text{ DJ Degree}) \times 30 \text{ DJ Degree} = 2/3$$

Next, the relationship between the hypotenuse and the adjacent side in DJ Trigonometry is applied:

$$K = O \sqrt{(1 + (\text{DJ Measure})^2)}$$

Substituting the known values,

$$K = 15 \sqrt{(1 + (2/3)^2)}$$

$$K = 15 \sqrt{(1 + 4/9)}$$

$$K = 15 \sqrt{(13/9)}$$

$$K = 15 \times (\sqrt{13} / 3)$$

$$K = 5\sqrt{13} \approx 18.03 \text{ meters}$$

Thus, according to DJ Trigonometry, the hypotenuse of the right-angled triangle is approximately 18.03 meters. In this example, the hypotenuse is calculated using only the DJ Angle, the corresponding DJ Measure, and the hypotenuse-adjacent side relationship formula. No trigonometric functions such as sine, cosine, or tangent, and no circular concepts such as radians or π are used. The result is obtained solely through basic arithmetic operations and square roots. This example clearly demonstrates that the hypotenuse-adjacent side relationship in DJ Trigonometry provides a simple, direct, and effective method for practical calculations.

6.3 Real-life Example 3

(iii) Example: Determination of the Hypotenuse (K) Using DJ Trigonometry.

(Application of the Hypotenuse-Opposite Side Relationship)

Suppose a right-angled triangle is formed. The length of the opposite side (P) corresponding to a given DJ Angle is known, and the objective is to determine the hypotenuse (K) of the triangle. First, the DJ Angle toward the object is measured using a DJ Protractor. Let the measured angle be

DJ Angle = 30 DJ Degree.

Now assume that:

Opposite side (P) = 10 meters.

According to DJ Trigonometry, the DJ Measure is first obtained from the given DJ Angle.

$$\text{DJ Measure} = (1/45 \text{ DJ Degree}) \times \text{DJ Angle}$$

$$\text{DJ Measure} = (1/45 \text{ DJ Degree}) \times 30 \text{ DJ Degree} = 2/3$$

Next, the relationship between the hypotenuse and the opposite side in DJ Trigonometry is applied:

$$K = P \sqrt{(1 + 1 / (\text{DJ Measure})^2)}$$

Substituting the known values,

$$K = 10 \sqrt{(1 + 1 / (2/3)^2)}$$

$$K = 10 \sqrt{(1 + 1 / (4/9))}$$

$$K = 10 \sqrt{(1 + 9/4)}$$

$$K = 10 \sqrt{(13/4)}$$

$$K = 10 \times (\sqrt{13} / 2)$$

$$K = 5\sqrt{13} \approx 18.03 \text{ meters}$$

Thus, according to DJ Trigonometry, the hypotenuse of the right-angled triangle

is approximately 18.03 meters.

In this example, the hypotenuse is calculated using only the DJ Angle, the corresponding DJ Measure, and the hypotenuse–opposite side relationship formula. No trigonometric functions such as sine, cosine, or tangent, and no circular concepts such as radians or π are used. The result is obtained solely through basic algebraic operations and square-root evaluation.

This example clearly demonstrates that the hypotenuse–opposite side relationship in DJ Trigonometry provides a simple, direct, and effective approach for practical calculations.

6.4 Real-life Example 4

Formulae Used in Example 4

Radius of a square

If the side length of a square is a , then the radius r of the square is given by

$$r = a / 2$$

Relationship between DJ Degree and DJ Measure

The conversion from DJ Degree to DJ Measure is given by

$$\text{DJ Measure} = (4 / 180) \times \text{DJ Degree}$$

Definition of DJ Measure

DJ Measure is defined as the ratio of the square perimeter segment to the radius of the square

$$\text{DJ Measure} = \text{Length of square perimeter segment} / r$$

Formula for square perimeter segment corresponding to a DJ Angle

$$\text{Length of square perimeter segment} = r \times \text{DJ Measure}$$

Application for a given DJ Angle

If DJ Angle = 120 DJ Degree, then

$$\text{DJ Measure} = (4 / 180) \times 120$$

$$\text{DJ Measure} = 8 / 3$$

Application for a given radius

If radius $r = 3$ units, then

$$\text{Length of square perimeter segment} = 3 \times (8 / 3)$$

$$\text{Length of square perimeter segment} = 8 \text{ units}$$

Conceptual Note

All the above formulae are based on square-oriented geometry. No circular arc, no π , no radian measure, and no classical trigonometric functions such as sine, cosine, or tangent are used. The DJ Measure directly connects angular information with linear perimeter length through simple proportional relations. This perimeter segment represents the proportional square boundary length corresponding to the given DJ Angle, not a single physical side of the square.

Importance of DJ Trigonometry in Mobile Calculators

In the present digital era, mobile calculators have become a common and easily accessible tool for mathematical computations. However, in classical trigonometry, the use of mobile calculators requires trigonometric functions such as sine, cosine, and tangent, along with careful handling of degree–radian modes and circular concepts. This often makes the calculation process complex and confusing, especially for beginners. In contrast, DJ Trigonometry provides a simple, direct, and calculator-friendly framework.

In DJ Trigonometry, angles are defined exclusively as DJ Angles, and their measurement is expressed in DJ Degree. From a given DJ Angle, the DJ Measure is

obtained directly. The DJ Measure is a proportional quantity that establishes a direct relationship between the sides of a right-angled triangle. This process does not involve circular angles, radians, or the constant π . A key feature of DJ Trigonometry is that the entire trigonometric analysis is based on four fundamental formulas expressed solely in terms of the DJ Measure. These formulas require only basic algebraic operations such as multiplication, division, and square roots. Consequently, there is no need to select or evaluate any trigonometric functions on a mobile calculator. Thus, in DJ Trigonometry, the mobile calculator functions merely as a simple numerical computation device, while the DJ Angle and DJ Measure provide a clear and controlled representation of angular and proportional relationships. This significantly simplifies the calculation process and reduces the possibility of conceptual or operational errors. Therefore, it can be concluded that DJ Trigonometry, particularly through the use of DJ Angle, is highly important, practical, and educationally effective for mobile calculator-based trigonometric calculations.

7. Objectives

The objectives of the present research are as follows:

- To introduce DJ Trigonometry as a new and independent trigonometric framework based on square-oriented and polygonal geometry rather than circular geometry.
- To define and formalize the concepts of DJ Measure, DJ Degree, and DJ Angle as alternative units for angular and proportional measurement in closed geometric figures.
- To develop a specialized geometric instrument, the DJ Protractor, for accurately measuring DJ Angles and visualizing angular relationships within polygon-based geometries.
- To establish the DJ Badwan Theorem as a fundamental proportional relationship between the sides of a right-angled triangle within the DJ Trigonometric system, without reliance on classical trigonometric functions.
- To demonstrate that trigonometric relationships can be applied consistently to closed figures such as triangles, squares, and rectangles using proportional reasoning instead of circular angle functions.
- To simplify trigonometric learning by reducing conceptual dependence on radians, circular arcs, and trigonometric tables, thereby improving mathematical clarity and accessibility for learners.
- To provide a coherent theoretical foundation that complements classical trigonometry while offering an alternative geometric perspective for educational and analytical applications.

8. Discussion

The DJ Trigonometry framework developed in this study should not be interpreted as a replacement for classical trigonometry, but rather as a complementary and alternative geometric framework. The purpose of this discussion is to analyze the extent to which the concepts of DJ Measure, DJ Degree, and DJ Angle are capable of expressing angle-side relationships in closed and polygon-based geometric structures in a clear and consistent manner. Classical trigonometry is fundamentally grounded in circular geometry, where angles are defined through arcs, radii, and radian measures. This framework is highly effective for circular, conical, and spherical geometries. However, when the focus shifts to polygonal or closed linear figures, circular

interpretations may appear indirect or conceptually abstract. In this context, DJ Trigonometry offers a different perspective by establishing angle-side relationships through square-based geometry and proportional reasoning. The principal theoretical contribution of this research is the DJ Badwan Theorem. The discussion clarifies that this theorem is not a reformulation of the classical tangent function, but instead represents an independent proportional relationship defined entirely within the DJ measurement system. By expressing the relationship between the sides of a right-angled triangle through the DJ Measure, the framework demonstrates that certain trigonometric relationships can be formulated without reliance on circular trigonometric functions.

An important aspect of DJ Trigonometry is the deliberate definition of the DJ Measure as a bounded proportional quantity, with values strictly less than one. This constraint is not merely a technical limitation, but a necessary condition for preserving the triangular nature of the geometric configuration. As the DJ Measure approaches unity, the geometric behavior transitions toward square-like symmetry, at which point trigonometric interpretation becomes less meaningful. This observation highlights the internal consistency and definitional clarity of the DJ framework. The numerical use of 360 in DJ Degree, analogous to the classical degree system, also warrants discussion. This similarity is not intended to suggest mathematical equivalence, but rather to maintain pedagogical continuity. The geometric foundations of DJ Degree and classical degree are fundamentally different, and this study explicitly avoids direct comparability between the two systems. By addressing this distinction at the theoretical level, the framework minimizes potential conceptual ambiguity. The findings of this study suggest that DJ Trigonometry may be particularly effective for educational and conceptual applications, where clarity and proportional intuition are essential. At the same time, it is acknowledged that broader acceptance of this framework will require further comparative analysis, formal mathematical validation, and application-based investigations. Overall, this discussion establishes DJ Trigonometry as a structured, bounded, and conceptually consistent framework that coexists with classical trigonometry while offering an alternative perspective for the geometric analysis of polygon-based and closed figures.

9. Conclusion

This research presents DJ Trigonometry as a new and independent trigonometric framework aimed at simplifying the understanding of angular and proportional relationships in closed geometric figures. The primary objective of this study was to develop an alternative approach to classical trigonometry that reduces conceptual complexity while preserving mathematical consistency. This objective has been achieved by introducing the concepts of DJ Measure, DJ Degree, DJ Angle, and a dedicated geometric instrument, the DJ Protractor, all of which are based on square-oriented geometry rather than circular constructions.

Unlike traditional trigonometry, which relies heavily on circular angle measurement and trigonometric functions, DJ Trigonometry establishes direct proportional relationships between sides and angles. The DJ Badwan Theorem plays a central role in this framework by providing a straightforward method to determine side lengths in a right-angled triangle using proportional reasoning, without invoking sine, cosine, or tangent functions. This contributes significantly to improving conceptual clarity, especially at the foundational level of mathematical learning. Furthermore, the

proposed system demonstrates that angular relationships in closed and polygonal figures can be analyzed consistently without dependence on circular arcs, radians, or π . By adopting a square-based measurement structure, DJ Trigonometry extends trigonometric reasoning to geometric forms such as squares, rectangles, and general triangles in a unified manner. The outcomes of this research suggest that DJ Trigonometry has strong potential as an educational and pedagogical tool, offering an intuitive pathway for learners to understand geometric proportionality. At the same time, the framework provides a mathematically coherent alternative perspective that complements, rather than contradicts, classical trigonometry. In conclusion, DJ Trigonometry fulfills its stated objectives by presenting a simplified, proportional, and non-circular approach to trigonometric analysis. This work lays a conceptual foundation for further theoretical refinement, mathematical validation, and possible future applications in geometry education and applied mathematical modeling.

Compliance with Ethical Standards***Ethical Approval***

This research is purely theoretical and mathematical in nature. It does not involve any human participants, animals, clinical experiments, surveys, or personal data. Therefore, ethical approval from an institutional review board or ethics committee was not required.

Consent to Participate

Not applicable. This study does not involve human subjects.

Consent for Publication

Not applicable. No personal, clinical, or identifiable data of any individual has been used in this manuscript.

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Conflict of Interest

The author declares that there is no conflict of interest regarding the publication of this paper. The research was conducted independently and without any external influence.

Author Contributions

This manuscript is a single-author work. The author solely conceived the idea, developed the theoretical framework, performed the mathematical analysis, designed the DJ Trigonometric concepts, and wrote and revised the entire manuscript.

Data Availability Statement

All data analyzed during this study are included within the manuscript and are available from the corresponding author upon reasonable request.

Code Availability

Not applicable. No computational code or software was used or generated in this study.

References

- [1] Euclid, *Elements, Book I & Book VI*, Ancient Greek Mathematical Treatise on Geometry and Proportionality, translated editions, Dover Publications.

- [2] Boyer, C. B., and U. C. Merzbach, *A History of Mathematics*, 2nd ed., Wiley, 2011.
- [3] Katz, V. J., *A History of Mathematics: An Introduction*, 3rd ed., Pearson Education, 2009.
- [4] Thomas, G. B., M. D. Weir, and J. Hass, *Thomas' Calculus*, 14th ed., Pearson. Sections on Trigonometry and Angle Measurement.
- [5] Stewart, J., *Calculus: Early Transcendentals*, Cengage Learning. Chapters on Trigonometric Functions and Geometric Interpretation.
- [6] Needham, T., *Visual Complex Analysis*, Oxford University Press, 1997. Discussion on geometric interpretation of angles and rotations.
- [7] Posamentier, A. S., *Challenging Problems in Geometry*, Dover Publications. Concepts of polygonal geometry and proportional reasoning.
- [8] Courant, R., and H. Robbins, *What Is Mathematics?*, Oxford University Press, 1996. Foundational ideas on geometry and mathematical abstraction.
- [9] Maor, E., *Trigonometric Delights*, Princeton University Press, 2004. Historical and conceptual development of trigonometry.
- [10] Kline, M., *Mathematical Thought from Ancient to Modern Times*, Oxford University Press, 1990.
- [11] Hilbert, D., *Foundations of Geometry*, Open Court Publishing. Axiomatic treatment of geometry.
- [12] National Council of Teachers of Mathematics (NCTM), *Principles and Standards for School Mathematics*, NCTM Publications. Relevance to simplified and alternative mathematical frameworks.
- [13] Polya, G., *How to Solve It*, Princeton University Press. Problem-solving and proportional reasoning in mathematics.
- [14] Stillwell, J., *Geometry of Surfaces*, Springer. Foundational geometric structures and measurement systems.
- [15] Badwan, D., "DJ Trigonometry: A Square-Based Proportional Framework for Closed Geometric Figures," Original research contribution (present work).

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